“Solar Energy” WS 2014/2015

Lecture 4: *p–n* Junction Diodes and Ideal Solar Cell Operation

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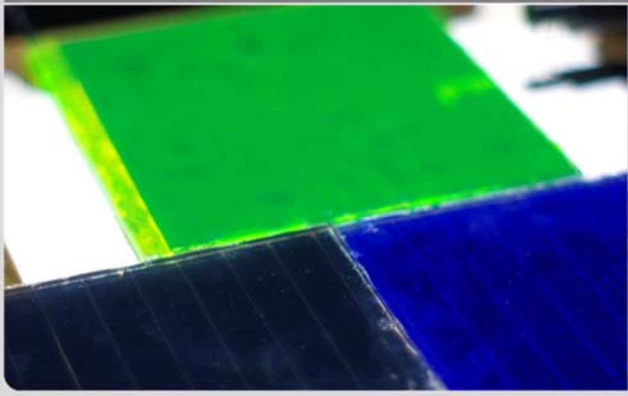
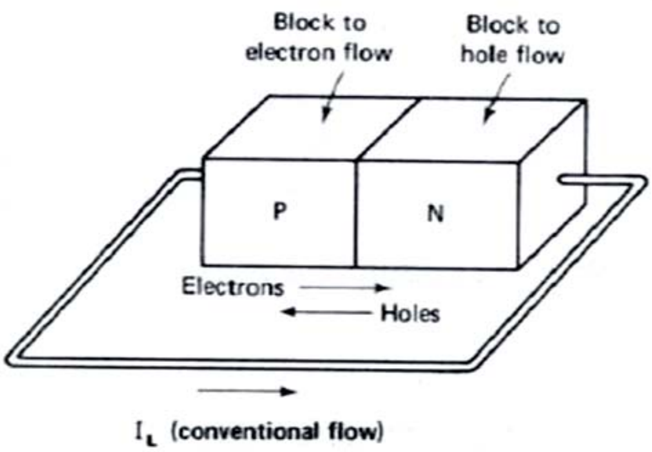
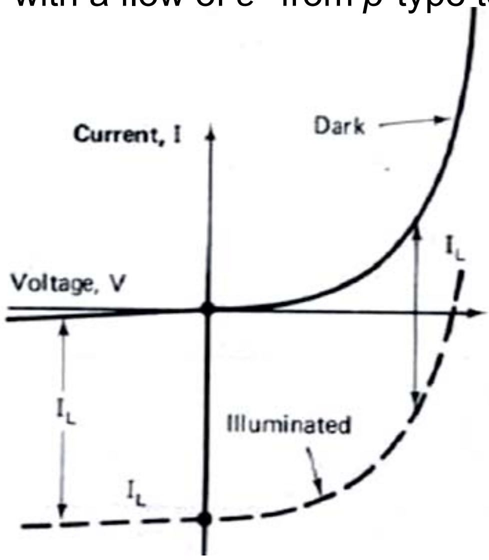


p–n Junction Diodes

Most common solar cells are just large diodes, formed by making a junction between *n*- and *p*-type material.

Basic requirement of electronic asymmetry: *n*-type regions have large e^- densities but small h^+ densities $\Rightarrow$ e^- flow easily but h^+ find it difficult. When illuminated, excess e^--h^+ pairs are generated, with a flow of e^- from *p*-type to *n*-type region $\Rightarrow$ current flow in the lead!





Source: Green, “Solar Cells”, Prentice Hall 1998

Fermi level revisited

Effective density of states in E_c given by:

$$n = N_c e^{((E_F - E_c)/kT)}$$

and likewise the number of h^+ in E_v

$$p = N_v e^{((E_v - E_F)/kT)}$$

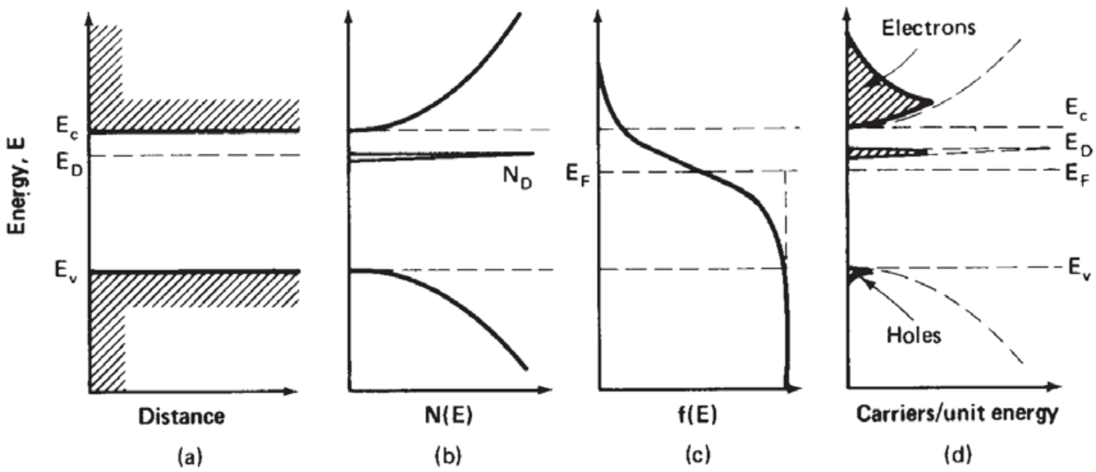
Hence, the Fermi level in an undoped semiconductor lies close to the midgap, being offset by differences in the effective density of states in E_c and E_v

$$E_F = \frac{E_c + E_v}{2} + \frac{kT}{2} \ln\left(\frac{N_v}{N_c}\right)$$

Fermi level revisited

Band diagram of

(a) group IV semiconductor with group V substitutional impurity of density N_D per volume;
(b) energy density of allowed states;
(c) probability of occupation of these states;
(d) resulting energy distributions of e^- and h^+



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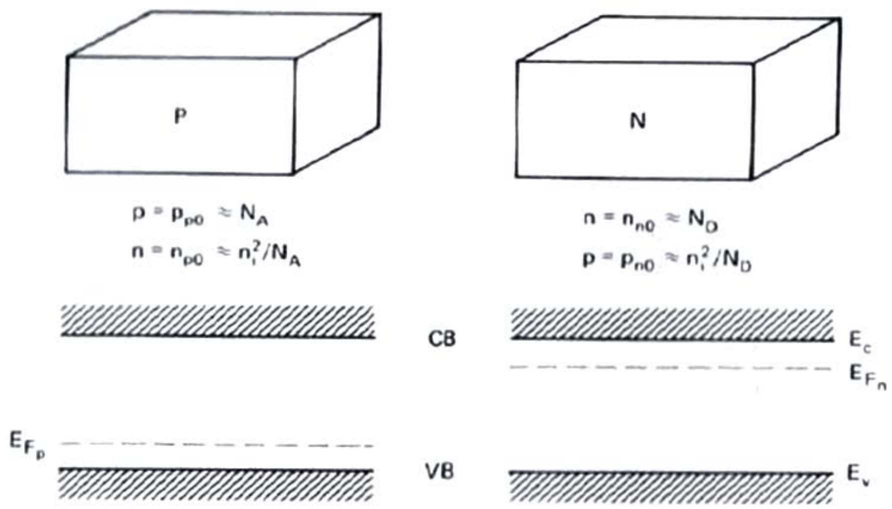
Hence, the Fermi level in an undoped semiconductor lies close to the midgap, being offset by differences in the effective density of states in E_c and E_v

$$E_F = \frac{E_c + E_v}{2} + \frac{kt}{2} \ln\left(\frac{N_v}{N_c}\right)$$

p–n Junction Diode Basics

Consider isolated pieces of *n*- and *p*-type material



Source: *Green, "Solar Cells", Prentice Hall 1998*

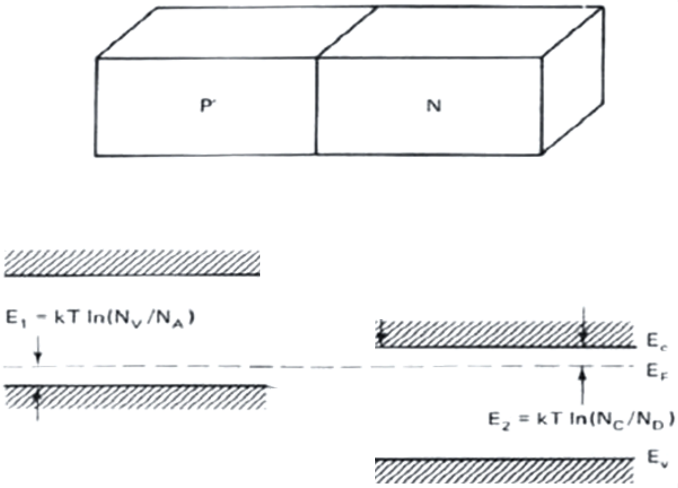
p–n Junction Diode Basics

Expected that excess e^- flow from region of high conc. (*n*-type) to regions of low conc. (*p*-type), and similarly for h^+

But, e^- leaving *n*-type side leave behind ionised donors (positive charge), and h^+ leaving *p*-type region will expose negative charges.

Exposed charges setup an electric field that oppose natural diffusion and an equilibrium situation will be reached **that has only one Fermi level**

Far enough away from the junction conditions are expected to remain unperturbed

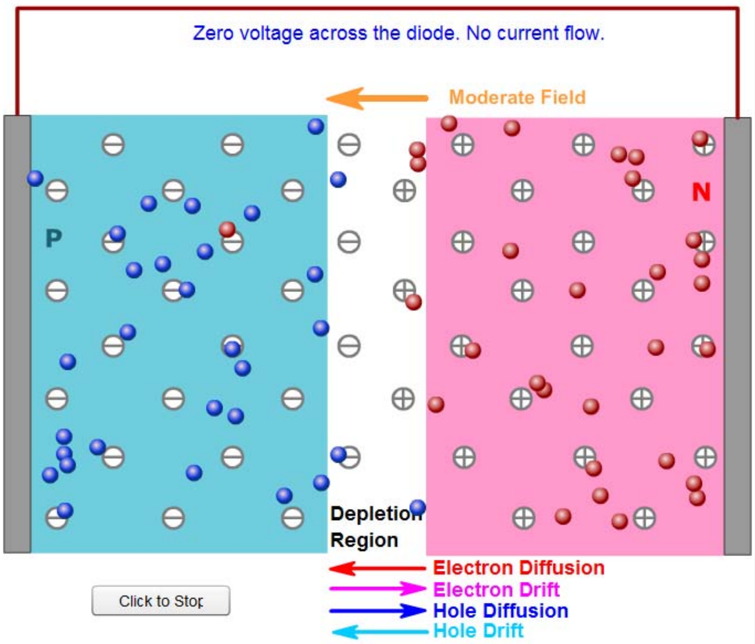


Source: Green, "Solar Cells", Prentice Hall 1998

p–n Junction Diode Basics

In equilibrium, the net current from the device is zero.

Electron drift current and electron diffusion current exactly balance out (same for holes)



Source: http://www.pveducation.org/pvcdrom/pn-junction/pn-junction-diodes

p–n Junction Diode Basics

But near the junction there is a transition region with a potential change ψ_0 occurs:

$$q\psi_0 = E_g - E_1 - E_2$$

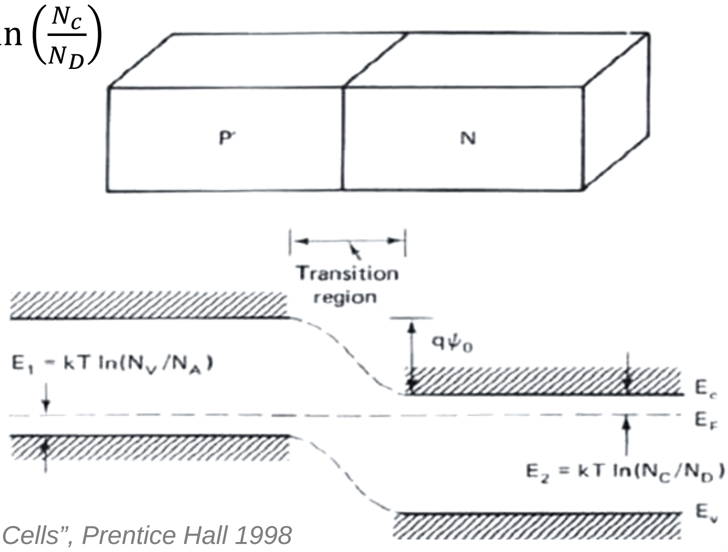
where

$$E_1 = kT \ln\left(\frac{N_v}{N_A}\right) \quad \text{and} \quad E_2 = kT \ln\left(\frac{N_c}{N_D}\right)$$

and hence

$$q\psi_0 = E_g - kT \ln\left(\frac{N_c N_v}{N_A N_D}\right)$$

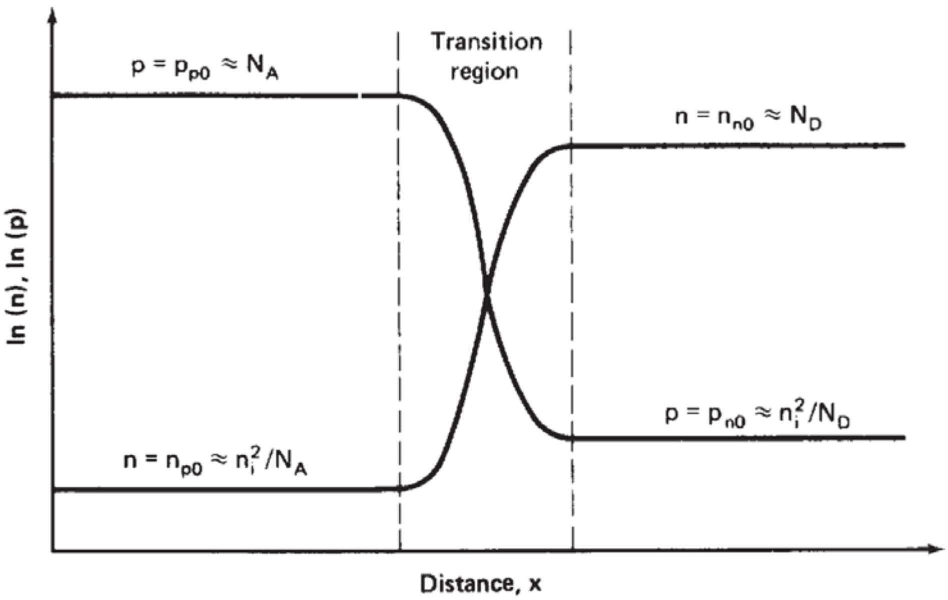
$$\psi_0 = \frac{kT}{q} \ln\left(\frac{N_A N_D}{{n_i}^2}\right)$$



Source: Green, “Solar Cells”, Prentice Hall 1998

p–n Junction Diode Basics

Distribution of carrier concentrations for previous figure – note logarithmic scale

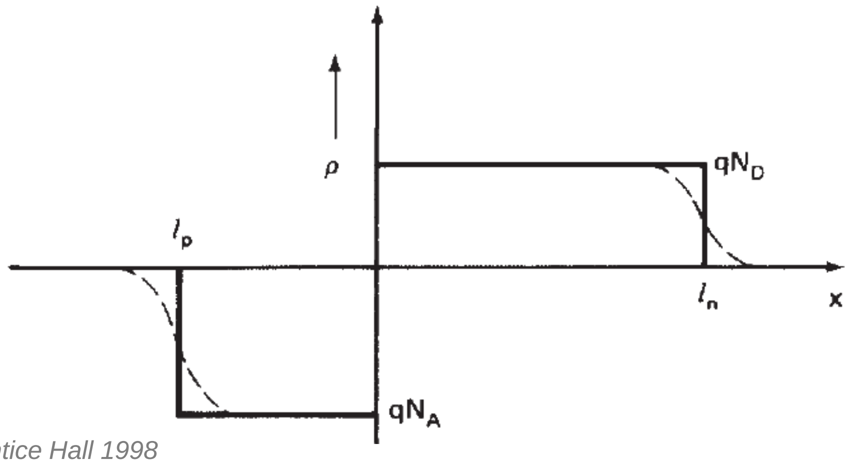


Source: Green, “Solar Cells”, Prentice Hall 1998

p–n Junction Diode Basics

Corresponding plot of space charge density ρ shown below (– – –)

Rapid change of ρ near edge of depletion region
$\Rightarrow$ Approximation 1: the depletion approximation (––––)
- Quasi-neutral regions where $\rho = 0$
- Depletion region where only contribution to ρ is from ionised dopants

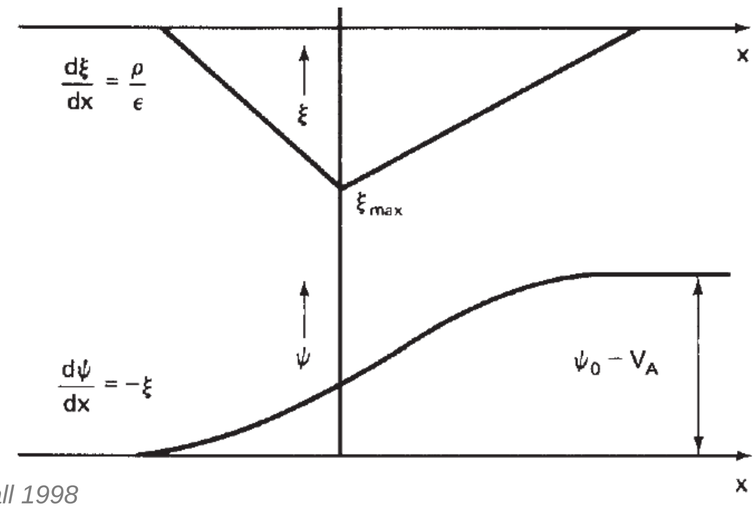


Source: *Green, "Solar Cells", Prentice Hall 1998*

p–n Junction Diode Basics

Can now determine following distributions across the depletion region:
- Electric field ξ - integration of the space charge density ρ
- Which is the negative gradient of the potential ψ

Here an applied voltage V_A is shown which changes the potential difference, thus that the potential across the transition region becomes ($\psi - V_a$)



Source: *Green, "Solar Cells", Prentice Hall 1998*

p–n Junction Diode Basics

Thus, maximum field strength in depletion region ξ_{max}, width of depletion region W and the distance that this extends either side of the junction l_p and l_n are:

$$\xi_{\max} = -\left[\frac{2q}{\epsilon}(\psi_0 - V_a)\bigg/\left(\frac{1}{N_A} + \frac{1}{N_D}\right)\right]^{1/2}$$

$$W = l_n + l_p = \left[\frac{2\epsilon}{q}(\psi_0 - V_a)\left(\frac{1}{N_A} + \frac{1}{N_D}\right)\right]^{1/2}$$

$$l_p = W\frac{N_D}{N_A + N_D} \qquad l_n = W\frac{N_A}{N_A + N_D}$$

p–n Junction Diode Basics

Presence of and width of depletion region can be measured.

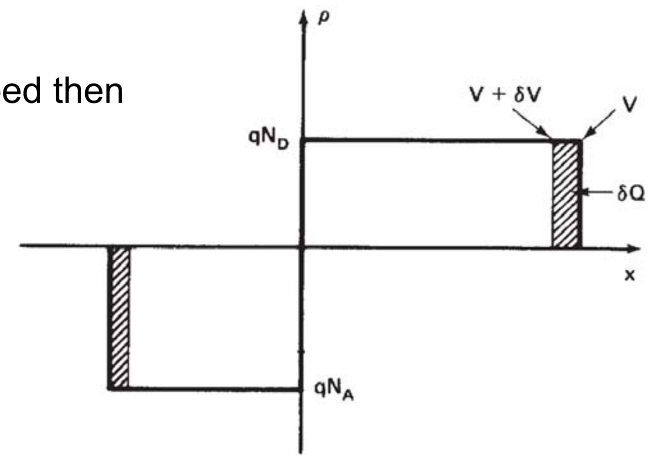
Change in applied voltage V_a causes a change in stored charge at the edges of the region. Identical situation to a parallel-plate capacitor with separation *W,* thus the depletion region capacitance is:

$$C = \frac{\epsilon A}{W}$$

and if one side of diode is heavily doped then

$$\frac{C}{A} = \left[\frac{q\epsilon N}{2(\psi_0 - V_a)}\right]^{1/2}$$

where N is the smaller of N_D or N_A



Source: Green, "Solar Cells", Prentice Hall 1998

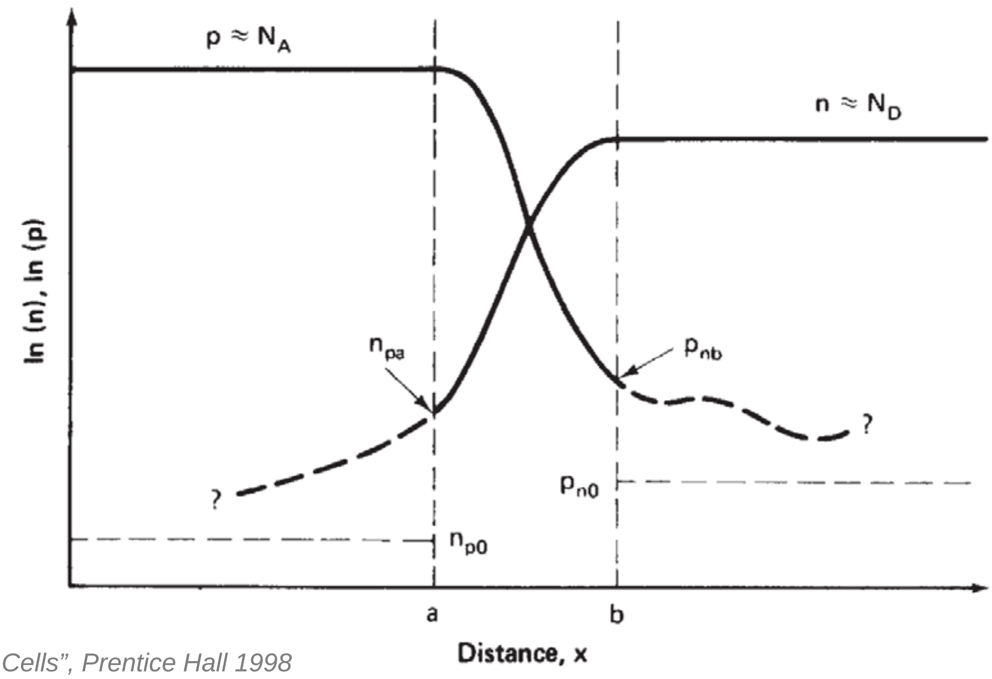
p–n Junction Diode Basics

Practical note:

In practice, the diode can be reverse biased and the depletion region capacitance ≈ total diode capacitance. Then measuring C as a function of reverse bias and plotting $1/C^2$ versus V_a allows N to be found

p–n Junction Diode Basics

Next we seek to determine the carrier concentrations at the edge of depletion region as a function of bias, $n_{p\,a}$ and $p_{n\,b}$



Source: *Green, "Solar Cells", Prentice Hall 1998*

p–n Junction Diode Basics

At zero bias we know that

$$p_{n\,b} = p_{n\,0} = p_{p\,0}\,\exp\left(-\frac{q\psi_0}{kT}\right) \approx \frac{n_i^2}{N_D}$$

$$n_{p\,a} = n_{p\,0} = n_{n\,0}\,\exp\left(-\frac{q\psi_0}{kT}\right) \approx \frac{n_i^2}{N_A}$$

Within depletion region have the highest electric field strengths and concentration gradients ⇒ current flow due to drift and diffusion large, but opposing (e.g. for holes)

$$J_h = q\mu_h p\xi - qD_h\,\frac{dp}{dx}$$

p–n Junction Diode Basics

At moderate bias we use approximation 2, that within depletion region

$$q\mu_h p\xi \approx qD_h\,\frac{dp}{dx}$$

and using Einstein's relation

$$\xi \approx \frac{kT}{q}\frac{1}{p}\frac{dp}{dx}$$

Integrating this over the depletion region gives

$$\psi_0 - V_a = -\frac{kT}{q}\ln p\Big|_a^b$$

$$= \frac{kT}{q}\ln\frac{p_{p\,a}}{p_{n\,b}}$$

p–n Junction Diode Basics

Rearranging this gives

$$p_{n\,b} = p_{p\,a} e^{-q\,\psi_0/kT} e^{q\,V_a/kT}$$

But from space-charge neutrality at point *a* and <u>assumption 3</u> – that we only consider cases where the no. of minority carriers << no. of majority carriers ($p_{p\,a}$ >> $n_{p\,a}$ and $n_{n\,a}$ >> $p_{n\,a}$) – we get

$$p_{pa} = N_A + n_{pa} \quad \text{(where } n_{pa} \text{ is small)}$$
$$\approx p_{p0} \approx p_{n0} e^{q\psi_0/kT}$$

p–n Junction Diode Basics

Hence

$$\boxed{\begin{aligned} p_{n\,b} &= p_{n0} e^{q\,V_a/kT} = \frac{n_i^2}{N_D} e^{q\,V_a/kT} \\ n_{p\,a} &= n_{p0} e^{q\,V_a/kT} = \frac{n_i^2}{N_A} e^{q\,V_a/kT} \end{aligned}}$$

This means that the minority carrier concentration at the edge of the depletion region increases with applied voltage – this process is known as minority carrier injection

N.B. Practically, we would know N_D, N_A and V_a

p–n Junction Diode Basics

If a uniformly doped region of the semiconductor is quasi-neutral, then minority carriers flow predominantly by diffusion – assumption 4

$$J_h = -qD_h \frac{dp}{dx} \qquad (n\text{-type quasi-neutral region})$$

$$J_e = qD_e \frac{dn}{dx} \qquad (p\text{-type quasi-neutral region})$$

Basically, the small number of minority carriers compared to majority carriers shields them from the effect of an electric field (see Green pages 71-72 for proof)

p–n Junction Diodes in the Dark

So, for the *n*-type side of the diode

$$J_h = -qD_h \frac{dp}{dx}$$

and continuity equation gives

$$\frac{1}{q}\frac{dJ_h}{dx} = -(U - G)$$

where we take the recombination rate as

$$U = \frac{\Delta p}{\tau_h}$$

where excess concentration of minority carrier holes, $\Delta p = p_n - p_{n0}$ (total hole concentration – equilibrium hole concentration) and τ_h is the minority carrier lifetime (constant)

p–n Junction Diodes in the Dark

Combining the above equations we get

$$D_h \frac{d^2 p_n}{dx^2} = \frac{p_n - p_{n0}}{\tau_h} - G$$

But in dark G = 0. Also $d^2 p_{n0}/x^2$ = 0. Thus simplifies to

$$\frac{d^2 \Delta p}{dx^2} = \frac{\Delta p}{L_h^2}$$ (remember $L_h = \sqrt{D_h \tau_h}$)

which has the general solution

$$\Delta p = Ae^{x/L_h} + Be^{-x/L_h}$$

p–n Junction Diodes in the Dark

The constants A and B are found by applying the two boundary conditions:

1. At $x = 0$, $p_{nb} = p_{n0} e^{qV/kT}$.
2. p_n finite as $x \to \infty$. Therefore, $A = 0$.

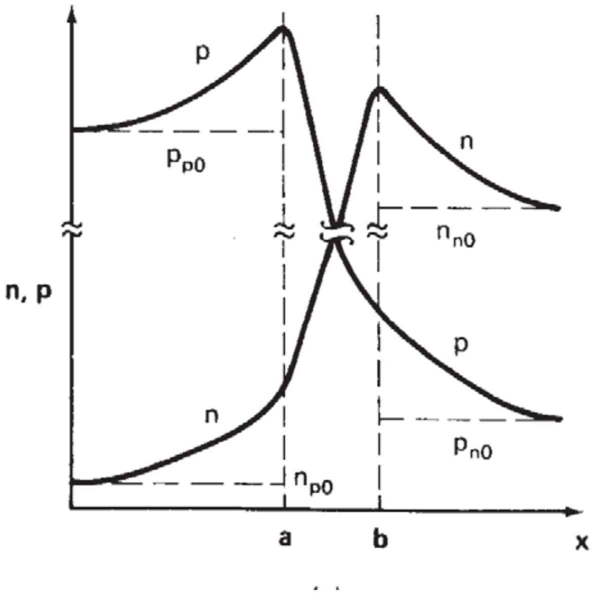
and gives the solution

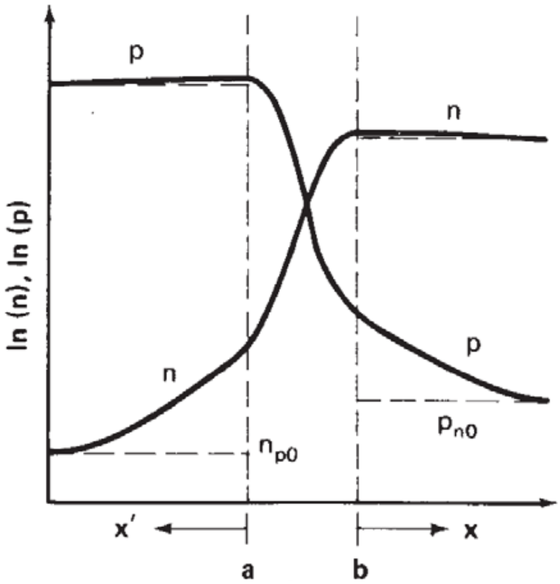
$$p_n(x) = p_{n0} + p_{n0}[e^{qV/kT} - 1]\, e^{-x/L_h}$$

$$n_p(x') = n_{p0} + n_{p0}[e^{qV/kT} - 1]\, e^{-x'/L_e}$$

Plotted in the next slide on both logarithmic and linear scales.

p–n Junction Diodes in the Dark





Source: *Green, "Solar Cells", Prentice Hall 1998*

p–n Junction Diodes in the Dark

Once carrier distributions are known, calculating current flow easy. On the *n*-type side

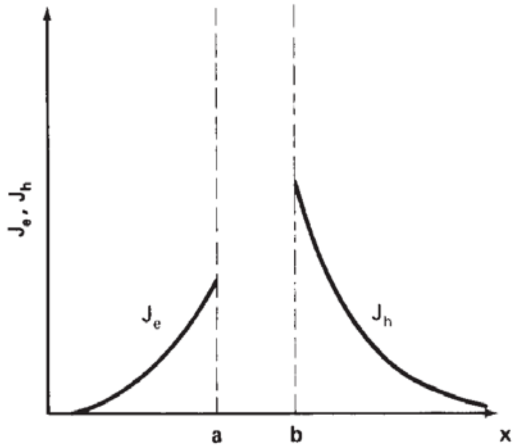
$$J_h = -qD_h \frac{dp}{dx}$$

And substituting in the previous solution for $p_n(x)$ gives

$$J_h(x) = \frac{qD_h p_{n0}}{L_h}\left(e^{qV/kT} - 1\right)e^{-x/L_h}$$

and similarly for *p*-type side

$$J_e(x') = \frac{qD_e n_{p0}}{L_e}\left(e^{qV/kT} - 1\right)e^{-x'/L_e}$$



Source: *Green, "Solar Cells", Prentice Hall 1998*

p–n Junction Diodes in the Dark

In order to calculate total current flow it is necessary to know e^- and h^+ components at same point

For current flows in depletion region the continuity equations tell us

$$\frac{1}{q}\frac{dJ_e}{dx} = U - G = -\frac{1}{q}\frac{dJ_h}{dx}$$

Thus magnitude of change in current across the depletion region is

$$\delta J_e = |\delta J_h| = q\int_{-W}^{0}(U - G)\,dx$$

Since W is very small, approximation 5 is that this integral is negligible so $\delta J_e = |\delta J_h| = 0$ and it follows that J_e and J_h are constant across the depletion region

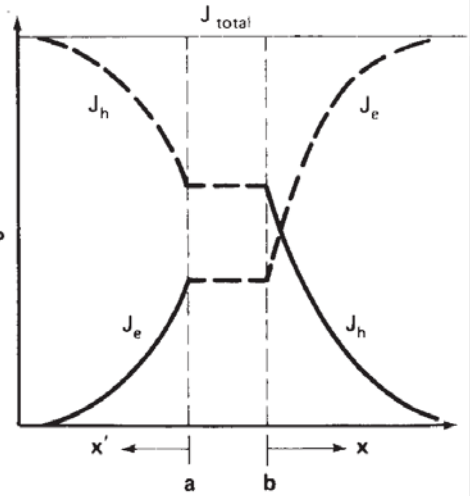
Source: Green, "Solar Cells", Prentice Hall 1998

p–n Junction Diodes in the Dark

Thus, total current can now be found

$$J_{\text{total}} = J_e\big|_{x'=0} + J_h\big|_{x=0}$$

$$= \left(\frac{qD_e n_{p0}}{L_e} + \frac{qD_h p_{n0}}{L_h}\right)(e^{qV/kT} - 1)$$



We have now derived the **ideal diode equation**

$$I = I_0(e^{qV/kT} - 1)$$

Where the saturation current I_0 is then

$$\boxed{I_0 = A\left(\frac{qD_e n_i^2}{L_e N_A} + \frac{qD_h n_i^2}{L_h N_D}\right)}$$

For a diode of cross-sectional area A

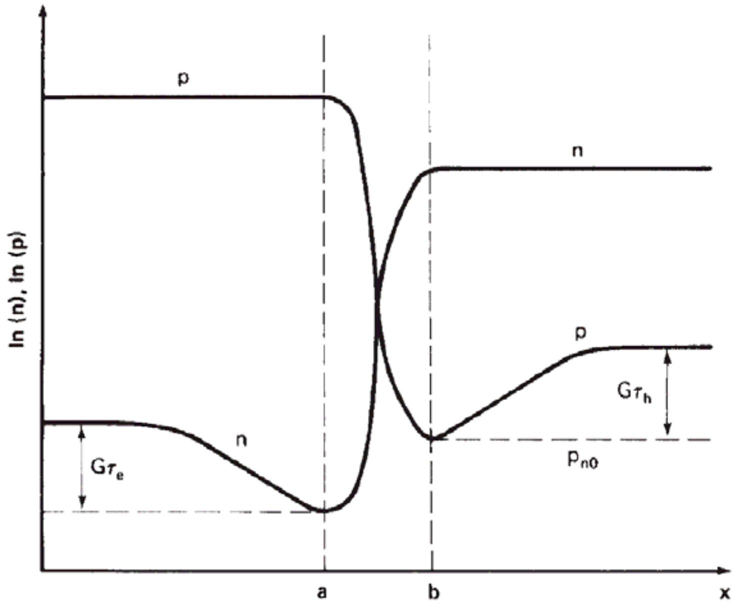
Source: Green, "Solar Cells", Prentice Hall 1998

Illuminated *p–n* Junction Diodes

Here the generation of rate of e^--h^+ pairs via illumination is assumed to be constant throughout the device (remember best approximation to this in reality is light with a similar energy to the bandgap of the solar cell)

This equation will be derived during next tutorial #2

Increase in minority carrier concentrations looks like this:



Source: Green, “Solar Cells”, Prentice Hall 1998

Illuminated *p–n* Junction Diodes

The current-voltage (I-V) characteristics of our illuminated diode are now:

$$I = I_0\,(e^{qV/kT} - 1) - I_L$$

where I_0 has the same value as before and I_L equals

$$I_L = qAG(L_e + W + L_h)$$

States that the light-generated current = value if all carriers generated within the depletion region – and within a minority diffusion length of it – were to contribute to it ⇒ this defines the “active” collection area of the solar cell

Illuminated *p–n* Junction Diodes

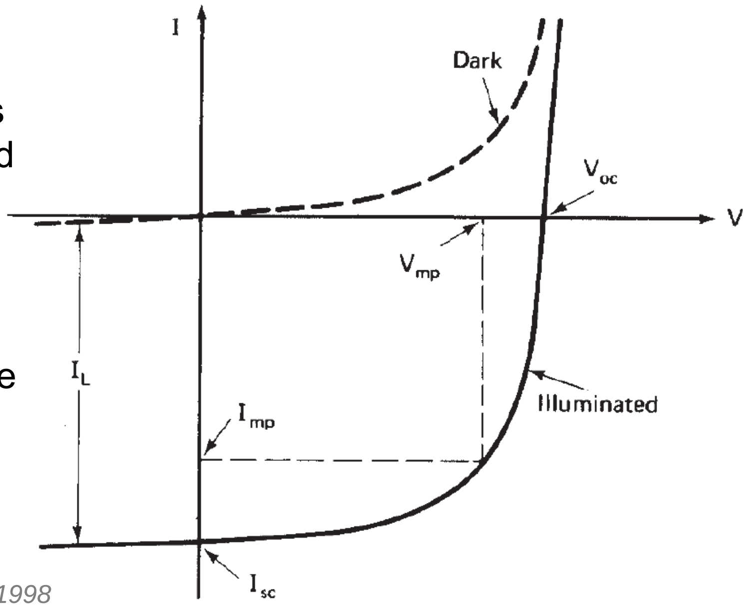
The current-voltage (I-V) characteristics of our illuminated diode are now:

$$I = I_0\,(e^{qV/kT} - 1) - I_L$$

as plotted here.

N.B. Illuminated characteristics are merely the dark ones shifted down by a current I_L.

This is in the fourth quadrant of the I-V curve as this is where power can be extracted from the diode



Source: Green, "Solar Cells", Prentice Hall 1998

p–n Junction Diodes Recap

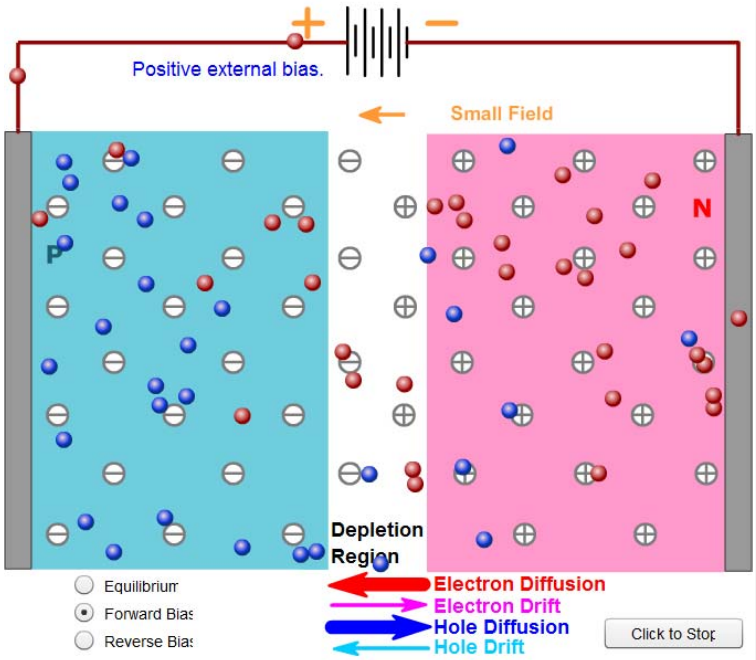
Diode is <u>forward biased</u> via application of voltage across the device such that the electric field at the junction is reduced (+V to the *p*-type material and a –V to *n*-type material)

Since the resistivity of depletion region >> remainder of device (due to the limited number of carriers in the depletion region) ⇒ nearly all of applied electric field is dropped across the depletion region

For realistic devices, built-in electric field is always larger than applied field, thus reducing the net electric field in the depletion region. Reducing the electric field ⇒ reduces the barrier to diffusion of carriers from one side of the junction to the other ⇒ increases diffusion current.

N.B. drift current remains essentially unchanged since it depends on the number of carriers generated within a diffusion length of the depletion region

p–n Junction Diodes Recap



Source: http://www.pveducation.org/pvcdrom/pn-junction/bias-of-pn-junction

p–n Junction Diodes Recap

Increased diffusion causes minority carrier injection at edge of depletion region ⇒ carriers move away from junction due to diffusion and eventually recombine with majority carrier

Majority carrier is supplied from external circuit and hence net current flows under forward bias.

Without recombination, minority carrier concentration would reach a new higher equilibrium concentration and the diffusion of carriers across junction would cease ⇒ hence diffusion current which flows in forward bias is a recombination current

The higher the rate of recombination events, the greater the current which flows across the junction. The dark saturation current (I_0) is an extremely important parameter which differentiates one diode from another and is a measure of the recombination in a device

p–n Junction Diodes Recap

In reverse bias, V is applied across the device and electric field at the junction increases ⇒ higher electric field in the depletion region decreases the probability that carriers can diffuse from one side of the junction to the other, hence the diffusion current decreases

A small increase in the drift current is experienced due to the small increase in W, but this is essentially a second-order effect in silicon solar cells.

However, in many thin film solar cells where the entire solar cell has a thickness of ~2W ⇒ change in W with voltage has a large impact on cell operation

Solar Cell Output Parameters

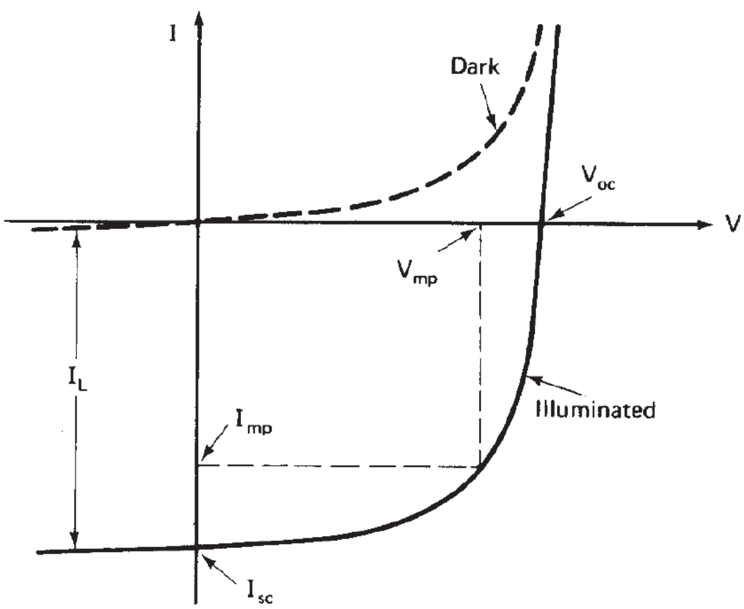
From the I-V curve, we can note a number of parameters that are used to characterise solar cells.

I_{sc} = short-circuit current (mA) which is ideally equal to I_L

V_{oc} = open-circuit voltage (mV)

which is found by setting I = 0 (no current flow at open circuit) in previous equation and rearraging

$$V_{oc} = \frac{kT}{q} \ln\left(\frac{I_L}{I_0} + 1\right)$$



Solar Cell Output Parameters

As well there is the current and voltage at the maximum power point I_{mp} and V_{mp} . This is where we extract the greatest amount of power.

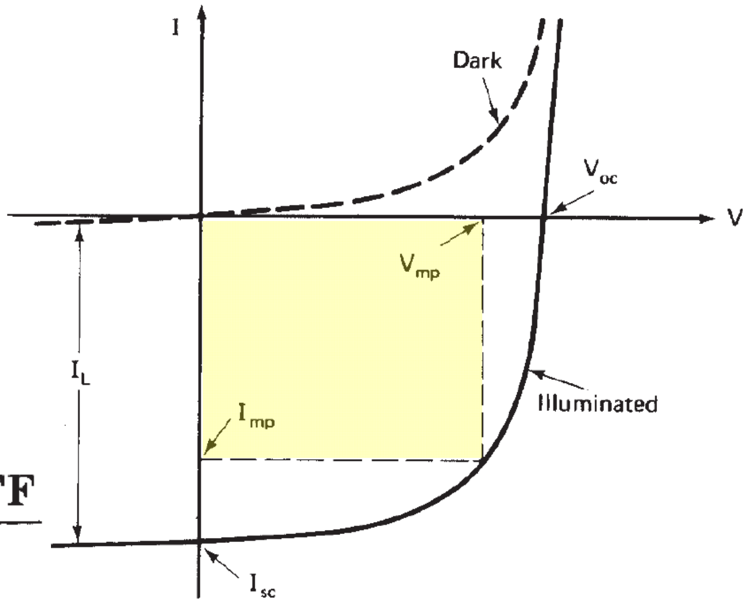
The fill factor (FF) is then defined as

$$FF = \frac{V_{mp} I_{mp}}{V_{oc} I_{sc}}$$

which is influenced by resistance losses in the device (more next lecture)

And finally the energy conversi efficiency is

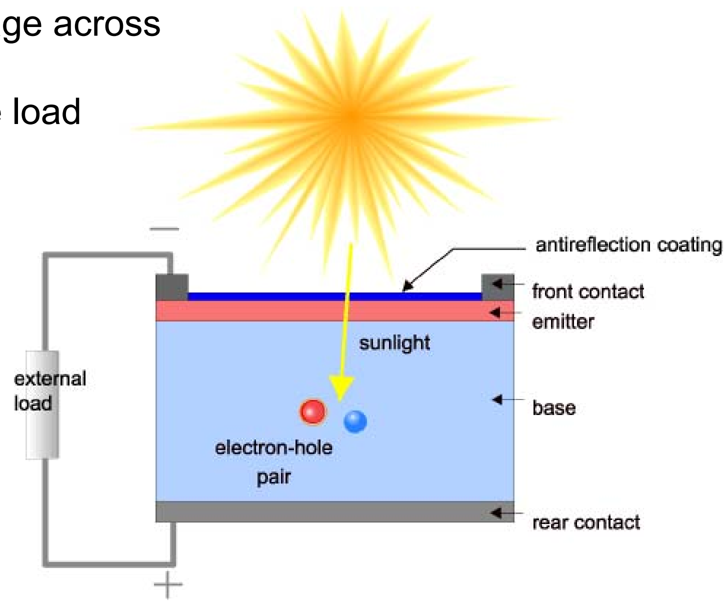
$$\eta = \frac{V_{mp} I_{mp}}{P_{in}} = \frac{V_{oc} I_{sc} FF}{P_{in}}$$



Solar Cell Operation

Basic steps in the operation of a solar cell are:

- the generation of light-generated carriers;
- the collection of the light-generated carries to generate a current;
- the generation of a large voltage across the solar cell; and
- the dissipation of power in the load and in parasitic resistances.

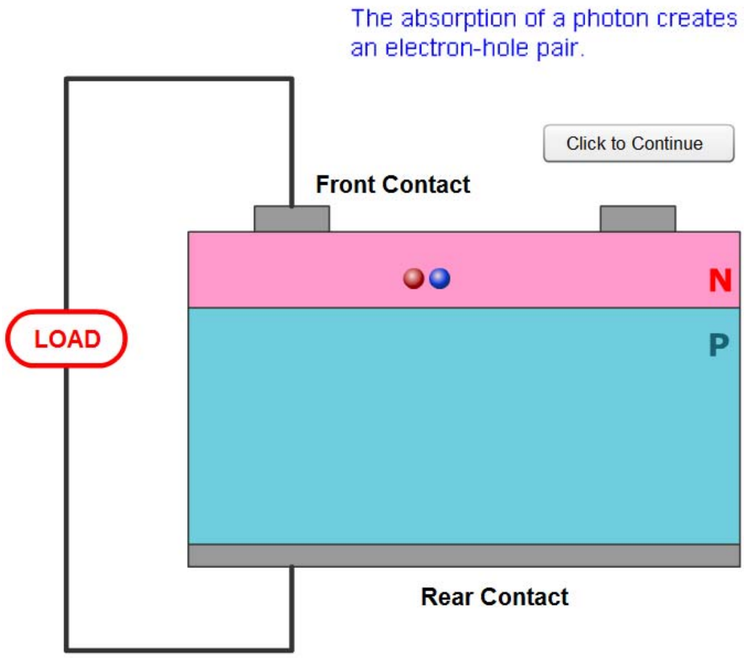


Source: http://www.pveducation.org/pvcdrom/solar-cell-operation/solar-cell-structure

Solar Cell Operation

Remember:

- Minority carriers are meta-stable and only exist, on average, for time = τ before recombining. If carrier recombines, then light-generated e^{-}-h^{+} pair is lost and no current or power can be generated
- Collection of these carriers facilitated by *p-n* junction, $\Rightarrow$ spatial separation of e^{-} and h^{+}. Carriers separated electric field existing at the *p-n* junction



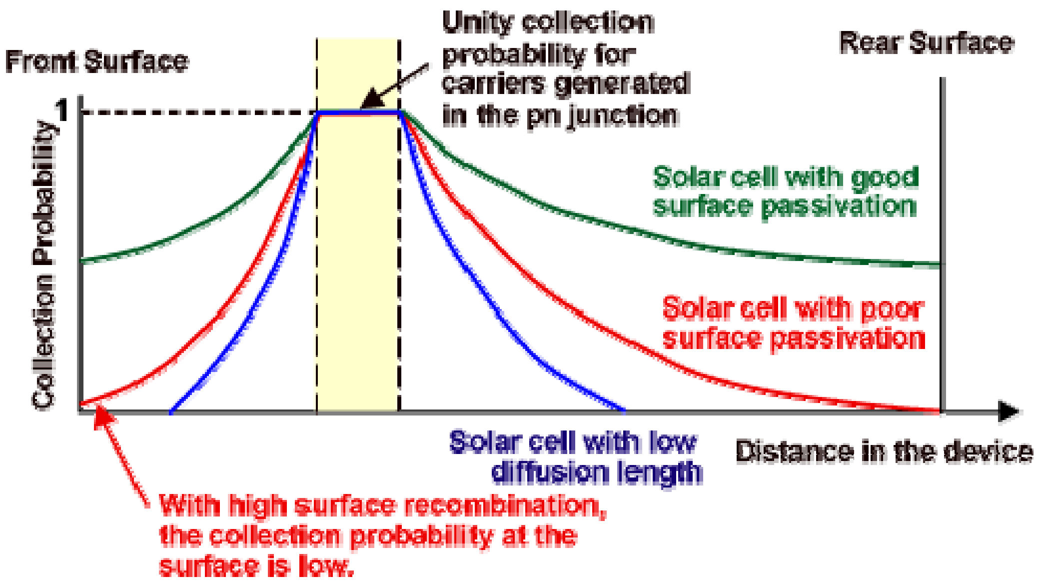
Source: http://www.pveducation.org/pvcdrom/solar-cell-operation/light-generated-current

Collection Probability

Collection probability that a carrier generated by light absorption in a certain region of the device will be collected by the *p-n* junction and $\therefore$ contribute to the light-generated current. Depends on:

- Distance that a carrier must travel c.f. diffusion length, *L*
- Surface properties of the device.
  - o Collection probability of carriers generated in depletion region is unity as e^{-}-h^{+} pair are quickly swept apart by electric field $\Rightarrow$ collected
  - o Away from junction, collection probability drops. For carriers generated > diffusion length away from junction $\Rightarrow$ collection probability quite low
  - o Similarly, if carrier is generated closer to a region such as a surface with higher recombination than the junction, then carrier likely to recombine. Hence, long diffusion lengths and good surface passivation important

Collection Probability



Source: http://www.pveducation.org/pvcdrom/solar-cell-operation/collection-probability

Light-Generated Current Density

Integration of collection probability *CP* and generation rate *G* over device thickness *W* of solar cell determines light-generated current density J_L

$$J_L = q\int_0^W G(x)CP(x)dx$$

$$= q\int_0^W \left[\int \alpha(\lambda)H_0 \exp(-\alpha(\lambda)x)\, d\lambda\right] CP(x)dx$$

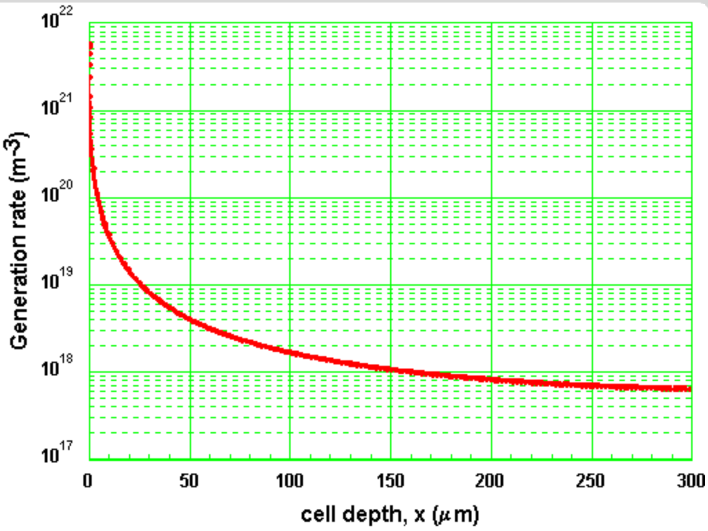
where:
q is the electronic charge;
W is the entire thickness of device;
$\alpha(\lambda)$ is the absorption coefficient;
H_0 is the number of photons at each wavelength

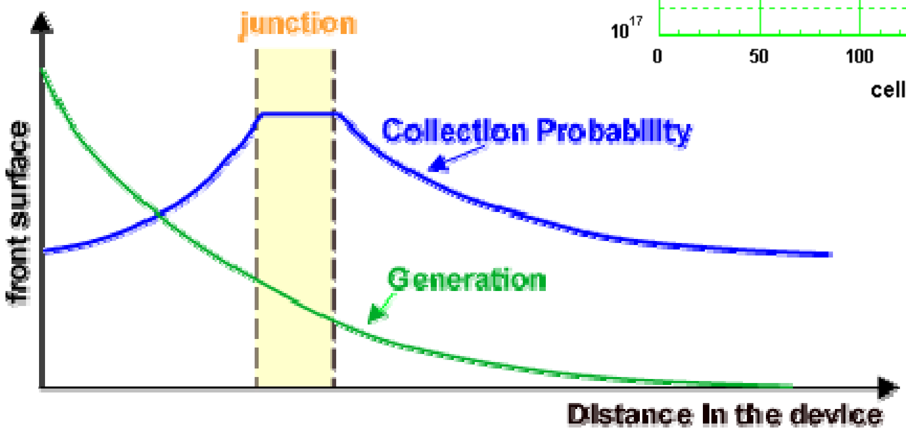
Plotted on next slide along with generation rate in silicon due to the AM1.5 solar spectrum

Light-Generated Current Density

Carrier generation is highest at front surface of solar cell

PV devices very sensitive to surface properties



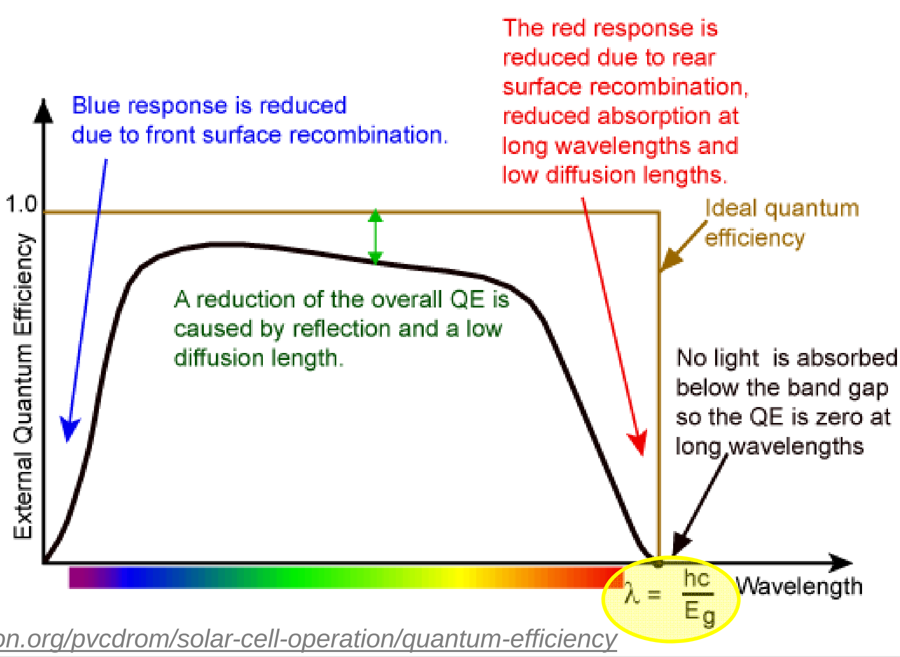


Source: http://www.pveducation.org/pvcdrom/solar-cell-operation/collection-probability

Quantum Efficiency

Quantum efficiency (QE) $\equiv \dfrac{\text{ratio of no. carriers collected by solar cell}}{\text{no. photons incident on solar cell at given wavelength}}$

- Unity QE achieved when all photons of certain λ are absorbed and resulting minority carriers collected
- QE for photons with energy < E_g is zero



Source: http://www.pveducation.org/pvcdrom/solar-cell-operation/quantum-efficiency

Quantum Efficiency

QE for practical solar cells reduced by same mechanisms that affect CP:

- blue light is absorbed very close to front surface ⇒ high front surface recombination will reduce the blue portion of QE
- green light is absorbed in the bulk of solar cell and now a low *L* will affect the *CP* ⇒ reduce QE in the green portion of spectrum.

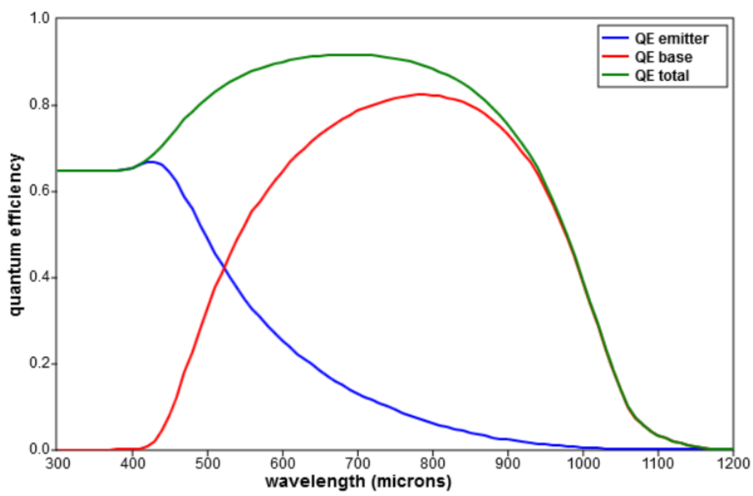
Note that:

- “External" quantum efficiency (EQE) of solar cell includes all optical losses such as transmission and reflection (i.e. ratio of e^{-}-h^{+} pairs collected per *incident* photon)
- Often useful to consider the “internal” quantum efficiency (IQE) which is the efficiency with which photons can generate collectable carriers after excluding optical losses (i.e. ratio of e^{-}-h^{+} pairs collected per *absorbed* photon)
- Practically, this is achieved by measuring the reflection and transmission of the solar cell and then correcting the EQE spectrum to obtain the IQE

Quantum Efficiency

Figure below shows how surface recombination and diffusion length affect the IQE of a solar cell

- emitter thickness 1 µm,
- base thickness is 300 µm,
- emitter diffusivity is 4 cm^2s^{-1}
- base diffusivity is 27 cm^2s^{-1}
- Front surface recombination velocity (SRV_{front}) = SRV_{rear} = 100 cm/s
- Emitter diffusion length = 1 µm
- Base diffusion length = 100 µm



Source: http://www.pveducation.org/pvcdrom/solar-cell-operation/quantum-efficiency

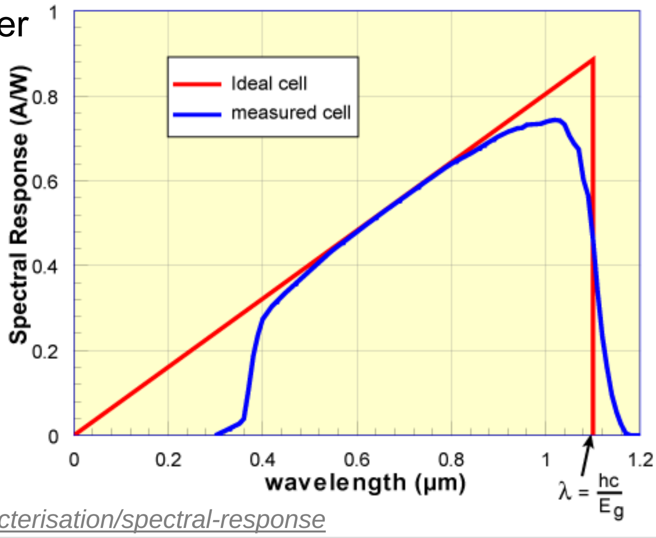
Spectral Response

Spectral response (SR) ≡ ratio of the current generated by solar cell to the power incident on the solar cell (common FoM for detectors)

1) Ideal SR is limited at long λ due to E_g
2) SR decreases at short λ as each photon possesses energy >> E_g
   ⇒ excess energy just results in lattice thermalisation losses (heat)
   ⇒ hence ratio of photons to power is reduced

- Two key loss mechanisms for solar cells made with single *p*-*n* junction
- SR is actually what is measured from solar cell ⇒ QE is then calculated via:

$$SR = \frac{q\lambda}{hc} QE$$

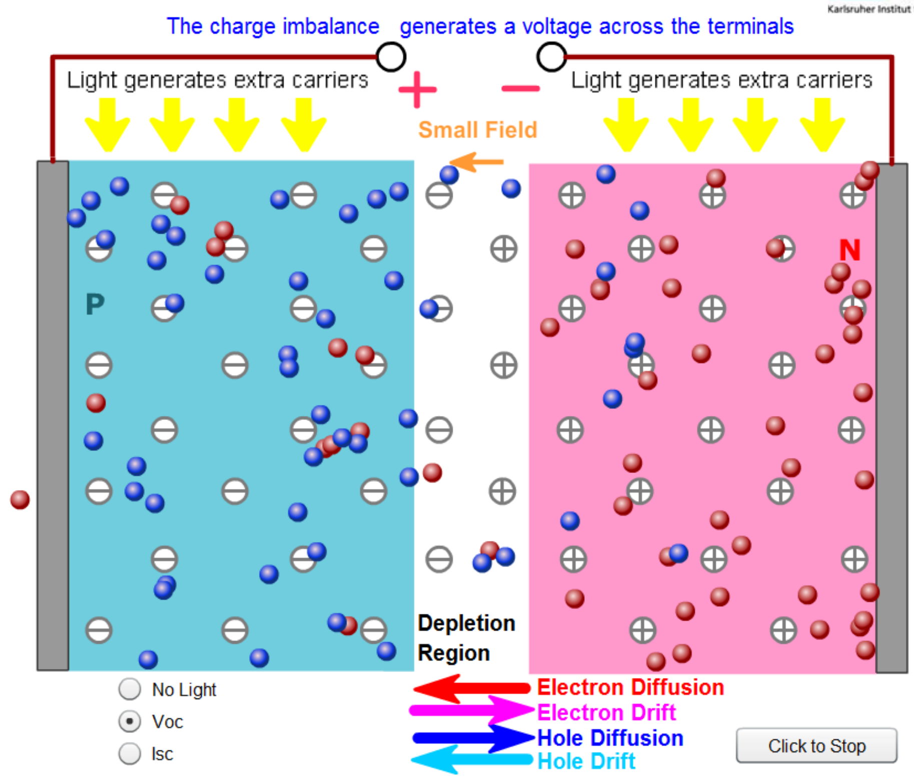


Source: http://www.pveducation.org/pvcdrom/characterisation/spectral-response

Overall: The Photovoltaic Effect

- Collection of light-generated carriers alone does not generate power
- Voltage is generated in a solar cell by the "photovoltaic effect“ ⇒ collection of light-generated carriers by the *p*-*n* junction causes a movement of e^- to the *n*-type side and h^+ to *p*-type side of junction
- Under short-circuit conditions, no build-up of charge as carriers exit the device as light-generated current
- However, if light-generated carriers are prevented from leaving the solar cell ⇒ collection of light-generated carriers causes an increase in no. e^- on *n*-type side of *p*-*n* junction (and similar increase in h^+ in *p*-type).
- Separation of charge creates an electric field at the junction (in opposition to that already existing), thereby reducing the net electric field. Since electric field represents a barrier to flow of forward-bias diffusion current, its reduction now increases the diffusion current ⇒ new equilibrium is reached in which a voltage exists across the *p*-*n* junction.
- At open-circuit, forward bias of junction increases to where I_L is exactly balanced by the forward bias diffusion current ⇒ net current is zero

Overall: The Photovoltaic Effect



Source: *http://www.pveducation.org/pvcdrom/solar-cell-operation/photovoltaic-effect*

Next Week

- **Tutorial #1 with Michael Oldenburg**

Lecture 5: Real Solar Cell Operation +
Design and Fabrication of Silicon Solar Cells